(b) Write the linear transformation corresponding to the matrix : **4**

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

5. Let $P_n(t)$ be the vector space of all polynomials of degree n. Let $D : P_3(t) \rightarrow P_2(t)$ be the linear transformation defined by $D(P(t)) = P'(t)$. Find the matrix D with respect to the standard basis $\{1, t, t^2, t^3\}$ and $\{1, t, t^2\}$. **8**

Unit III

6. (a) Let $T_1 : R^3 \rightarrow R^2$ s.t. $T_1(x, y, z) = (3x, 4y - z)$ and $T_2 : R^2 \rightarrow R^2$ s.t. $T_2(x, y) = (-x, y)$. Compute T_1T_2 and T_2T_1. **4**

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B.Sc. EXAMINATION, 2026

(Sixth Semester)

(Re-appear Only)

MATHEMATICS

Paper BM-362

Linear Algebra

Time : 3 Hours] [*Maximum Marks :* 40

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

1. (a) Define Subspace. **1**

(b) Define linear sum of subspaces. **1**

(c) Write vector $u = (2, -5, 4)$ as a linear combination of vectors $(1, -3, 2)$ and $(2, -1, -1)$ **2**

(d) Show that $T(x, y) = (x + y, 2x - 3y)$ is a linear transformation. **2**

(e) Define eigenvalue and eigen vectors. **2**

Unit I

2. (a) Prove that union of subspaces is a subspace iff one is contained in the other **4**

(b) Show that the set $(1, 2, 3)$, $(0, 1, 2)$, $(0, 0, 1)$ which is a subset of $V_3(\mathbf{R})$ spans the entire vector space V_3. **4**

3. (a) Expand the following set of vectors to form a basis of $\mathbf{R}^3$ $(1, 2, 3)$, $(2, -2, 0)$. **4**

(b) If W is a subspace of $V_3(\mathbf{R})$ generated by $(1, 0, 0)$, $(0, 0, 1)$, find V/W and its basis. **4**

Unit II

4. (a) Show that the transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1, x_2)$ is a L.t. and is onto but not one-one. **4**

(b) If B = (1, – 2, 3), (1, – 1, 1), (2, – 4, 7) is a basis of $\mathbf{R}^3$, then find the dual basis of B. **4**

7. (a) If T is an invertible operator and λ is an eigenvalue of T, then λ^{-1} is an eigen value of T^{-1}. **4**

(b) Prove that the minimal polynomial of a matrix or linear operator is unique. **4**

Unit IV

8. (a) State and prove Schwartz inequality. **4**

(b) Let V be an inner product space. Then prove that : **4**

$$||u + v|| \leq ||u|| + ||v||$$

9. Obtain an orthonormal basis with respect to the standard inner product for the subspace of $\mathbf{R}^3$ generated by (1, 0, 1), (2, 0, 1) and (0, 3, 4). **8**

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