Unit III

6. (a) Using rule of differentiation, find the derivative of $\cos^2(2z+3i)$. **4**

(b) Prove that real and imaginary parts of an analytic function satisfies Laplace's equation. **4**

7. (a) Construct analytic function of which real part is $u(x, y) = e^x (x\cos y - y\sin y)$. **4**

(b) Show that $f(z) = |z|^2$ is continuous everywhere but not analytic at any point. **4**

Unit IV

8. (a) Find the image of $|z-3i|=3$ under the mapping $w=\frac{1}{z}$. **4**

(b) Find the fixed points, normal form and nature of following Hobius transformation $w=\frac{3z-4}{z-1}$. **4**

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B. Sc. EXAMINATION, 2026

(Sixth Semester)

(Re-appear Only)

MATHEMATICS

Paper BM-361

Real and Complex Analysis

Time : 3 *Hours*] [*Maximum Marks :* 40

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Evaluate $\int_0^1 x^3(1-x)^{4/3}dx$. **2**

(b) Define fixed points for Mobius transformation. **2**

(c) Find Fourier co-efficient a_n for the function $f(x) = x$ in $[-\pi, \pi]$. **1**

(d) Find k so that function for $f(z) = r^2 \cos 2\theta + ir^2 \sin k\theta$ is analytic. **2**

(e) Define Critical Points. **1**

Unit I

2. (a) If $x = r\cos\theta, y = r\sin\theta, z = z$, then evaluate $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$. **4**

(b) Prove that : **4**

$$\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \, B(m+1, n+1).$$

3. (a) Show that : **4**

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}.$$

(b) Evaluate $\iint \sqrt{a^2 - x^2 - y^2}\, dxdy$ over the semicircle $x^2 + y^2 = ax$ in the positive quadrant. **4**

Unit II

4. (a) Expand $f(x) = x \sin x$ as a Fourier series in $(0, 2\pi)$. **4**

(b) Obtain Fourier series for the function

$$f(x) = \begin{cases} \pi x & 0 \le x \le 1 \\ \pi(2-x) & 1 \le x \le 2 \end{cases}. \quad \textbf{4}$$

5. (a) Obtain $f(x) = x$ is half range cosine series in $0 < x < 2$. **4**

(b) Find the Fourier series of x^2 in $(-\pi, \pi)$. Use Parseval's identity, show that

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots \quad \textbf{4}$$

9. (a) Show that unit circle $|z|=1$ is mapped onto $\left|z+\frac{1}{2}\right|=1$ under transformation $w=\frac{5-4z}{4z-2}$. **4**

(b) Find the image of upper half plane under transformation $w=\left|\frac{z-ic}{z+ic}\right|^2$ $c\in\mathbf{R}$. **4**

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