(b) Discuss the convergence of the series : **4**

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n}.x^{n}, x>0.$$

7. (a) Test the series $\sum_{n=1}^{\infty} ne^{-n^2}$ for convergence. **4**

(b) Using Cauchy's condensation test, discuss the convergence of the series : **4**

$$\sum_{n=1}^{\infty}\frac{1}{n(\log n)^{p}}.$$

Section IV

8. (a) If $\sum_{n=1}^{\infty} a_n$ is convergent and the sequence $< b_n >$ is monotonic and bounded, prove that $\sum_{n=1}^{\infty} a_n.b_n$ is convergent. **4**

No. of Printed Pages : 05 Roll No. .....

52276

B. Sc. EXAMINATION, 2026

(Fourth Semester)

(Re-appear Only)

MATHEMATICS

Paper BM-241

Sequence and Series

Time : 3 *Hours*] [*Maximum Marks :* 40

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Give an example of a set which is not the neighbourhood of any real number. **1**

(b) Test the convergence of the series $\sum_{n=1}^{\infty} \cos \frac{1}{n}$. **2**

(c) Show that $\langle (-1)^n \rangle$ is not a Cauchy Sequence. **2**

(d) Test the convergence of the series $\sum_{n=2}^{\infty} [\log (n+1) - \log n]$. **2**

(e) Give an example of a set having one limit point. **1**

Section I

2. (a) The intersection of an arbitrary family of closed sets is closed. **4**

(b) Show that $(A \cup B)' = A' \cup B'$, where $A \subseteq R$, $B \subseteq R$. **4**

3. (a) State and prove Bolzano-Weierstrass theorem. **4**

(b) The set A is closed iff $\bar{A} = A$. **4**

Section II

4. (a) State and prove Cauchy's first theorem on limits. **4**

(b) If a is any real number such that $0 < a < 1$, then $\lim_{n \to \infty} a^n = 0$. Prove. **4**

5. (a) Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} \right)^{\frac{1}{n}}$ is divergent. **4**

(b) Use Cauchy's general principle of convergence to show that the sequence $\left\langle \frac{n}{n+1} \right\rangle$ is convergent. **4**

Section III

6. (a) State and prove Raabe's test for the convergence of an inifite series. **4**

(b) Pove that the Cauchy product of the two series $3+\sum_{n=1}^{\infty}3^n$ and $-2+\sum_{n=1}^{\infty}2^n$ is absolutely convergent, even if both the given series are divergent. **4**

9. (a) Show that the infinite product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)$ ..... is convergent. **4**

(b) Discuss the convergence of the infinite product $\sum_{n=1}^{\infty}\left[1+\left(\frac{nx}{n+1}\right)^n\right], x>0$. **4**

❖❖❖❖❖❖❖

(b) Pove that the Cauchy product of the two series $3+\sum_{n=1}^{\infty}3^n$ and $-2+\sum_{n=1}^{\infty}2^n$ is absolutely convergent, even if both the given series are divergent. **4**

9. (a) Show that the infinite product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)$ ..... is convergent. **4**

(b) Discuss the convergence of the infinite product $\sum_{n=1}^{\infty}\left[1+\left(\frac{nx}{n+1}\right)^n\right], x>0$. **4**

❖❖❖❖❖❖❖