(b) Show that : **4**

$$\nabla^2\left(\frac{x}{r^2}\right) = -\frac{2x}{r^4}.$$

Unit III

6. (a) Derive an expressions for $\nabla\phi$ in orthogonal curvilinear coordinates. **4**

(b) Express the vector $x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical coordinates. **4**

7. (a) If $(r\theta\phi)$ are spherical coordinates, show that $\nabla\left(\frac{1}{r}\right) = \nabla \times (\cos\theta\nabla\phi)$. **4**

(b) Express the velocity $\vec{v}$ of a particle in cylinderical coordinates. **4**

Unit IV

8. (a) If $\vec{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$; evaluate $\iint_S \vec{f}.\vec{n}d\mathrm{S}$, where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. **4**

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B. Sc. EXAMINATION, 2026

(Second Semester)

(Re-appear Only)

MATHEMATICS

Paper BM-123

Vector Calculus

Time : 3 *Hours*] [*Maximum Marks :* 40

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Show that vectors $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - 7\hat{k}$ and $\vec{c} = 5\hat{i} + 6\hat{j} - 5\hat{k}$ are coplanar. **2**

(b) If $\vec{\omega}$ is a constant vector $\vec{v} = \vec{\omega} \times \vec{r}$, prove that $\text{div}\,\vec{v} = 0$. **1½**

(c) Determine the transformation from cyclindrical to rectangular co-ordinates. **1½**

(d) Define Line Integral. **1½**

(e) State Stoke's Theorem. **1½**

Unit I

2. (a) Show that : **4**

$$[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}\,\vec{b}\vec{c}].$$

(b) Find the set of vectors reciprocal to $\vec{a}$, $\vec{b}, \vec{a} \times \vec{b}$. **4**

3. (a) If $\vec{r}$ is a unit vector in the direction of $\vec{r}$, then prove that $\vec{r} \times \frac{d\vec{r}}{dt} = \frac{1}{r^2}\vec{r} \times \frac{d\vec{r}}{dt}$. **4**

(b) A particle moves along a curve $x = 3t^2$, $y = t^2 - 2t$, $z = t^3$. Find velocity and acceleration at $t = 2$. **4**

Unit II

4. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$, then prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$. **4**

(b) If $\phi(x, y, z) = x^2y + y^2x + z^2$, find grad ϕ at the point (1, 1, 1). **4**

5. (a) Find a, b, c so that the vector :

$$\vec{f} = (2x + 3y + az)\hat{i} + (bx + 2y + 3z)\,\hat{j} + (2x + cy + 3z)\,\hat{k}$$

is irrotational. **4**

(b) The acceleration of a particle at any time t is given by $e^t\hat{i} + e^{2t}\hat{j} + \hat{k}$. Find $\vec{V}$ given that $\vec{V} = \hat{i} + \hat{j}$ at $t = 0$. **4**

9. (a) State and prove Green's theorem. **4**

(b) Evaluate the Stokes' theorem :

$$\oint_C (e^x dx + 2ydy - dz$$

where C is the curve $x^2 + y^2 = 4$, $z = 2$. **4**

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