Unit III

6. (a) State and prove De-Moivre's Theorem for integers. **4**

(b) Prove that : **4**

$16 \sin^5 \theta = \sin 5\theta - 5 \sin 3\theta + 10 \sin \theta$.

7. (a) Express $\sin^6 \theta . \cos^2 \theta$ in a series of cosines of multiples of θ. **4**

(b) Prove that $\tanh \theta + \coth \theta$ is a periodic function with a period πi. **4**

Unit IV

8. (a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, prove that $xy + yz + zx = 1$. **4**

(b) Resolve $\log (1 + i)$ into real and imaginary parts. **4**

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B.Sc. EXAMINATION, 2026

(Second Semester)

(Re-appear Only)

MATHEMATICS

BM-121

Number Theory and Trigonometry

Time : 3 *Hours*] [*Maximum Marks :* 40

Before answering the question-paper, candidates must ensure that they have been supplied with correct and complete question-paper. No complaint, in this regard will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Solve the congruence $3x \equiv 5 \pmod 7$. **1.5**

(b) Show that $5^{10} - 3^{10}$ is divisible by 11. **1.5**

(c) Define Euler's ϕ function with example. **2**

(d) Find real part of cosh $(x + iy)$. **1.5**

(e) Define Logarithm of Complex Numbers. **1.5**

Unit I

2. (a) Find g.c.d. of 595 and 252 and express it in the form $m.595 + n.252$. **4**

(b) If a and m are co-prime, then prove that the linear congruence $ax \equiv b \pmod m$ has a unique solution. **4**

3. (a) Show that $n^7 - n$ is divisible by 42. **4**

(b) State and prove Fermat's Theorem. **4**

Unit II

4. (a) Using Gauss Lemma, show that 2 is a quadratic non-residue (mode 13). **4**

(b) State and prove Chinese Remainder Theorem. **4**

5. (a) Define Moebius function. State and prove Moebius Inversion formula. **4**

(b) Show that : **4**

$\phi(n) = \phi(n + 1) = \phi(n + 2)$ for $n = 5186$.

9. (a) If $\log \sin(\theta + i\phi) = \alpha + i\beta$, prove that : **4**

$$2\cos 2\theta = e^{2\phi} + e^{-2\phi} - 4e^{2\alpha}$$

and $\cos(\theta - \beta) = e^{2\phi} \cos(\theta + \beta)$

(b) Find the sum of the series : **4**

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \left(\frac{1}{2}\right)^2 \sin\alpha + \ldots\ldots \text{ to } \infty.$$

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9. (a) If $\log \sin(\theta + i\phi) = \alpha + i\beta$, prove that : **4**

$$2\cos 2\theta = e^{2\phi} + e^{-2\phi} - 4e^{2\alpha}$$

and $\cos(\theta - \beta) = e^{2\phi} \cos(\theta + \beta)$

(b) Find the sum of the series : **4**

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \left(\frac{1}{2}\right)^2 \sin\alpha + \ldots\ldots \text{ to } \infty.$$

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