Sr. No. 8022

M.Sc. (Maths), First Semester Examination, Dec 2016

Topology – I

Time: 3 Hours | Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) State and prove confinite topology on a non-empty set X. 6

   (b) Prove that in a top space (X,j), $\overline{A} = A \cup d(A) \forall A \subseteq X$. 10

2. (a) Prove that a family β of sets is a base for a topology for the set $X = U\{B: Bt\beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2, \exists\ a\ B \in \beta$ s.t. $x \in \beta \underline{c} B_1 \cap B_2$. 8

   (b) Characterize topology in terms of Kuratowsti closure operator. 8

Unit – II

3. (a) If f is a mapping of a top space X into another top space, then prove that f is continuous on X iff $f(c(E))\underline{C}C^*f(E)\forall E\underline{C}X$. 8

   (b) If a connected set C has a non-empty intersection with both a set E and the complement of E in a top space (X,j), then show that C has a non-empty entersection with the boundary of E. 8

4. (a) Define component of a space. Is componenet of a space necessarily open? 8

   (b) Prove that every protection $\pi_\lambda: X \to X_\lambda$ on a product space $X = \underset{\lambda}{\overset{\pi}{}} X_\lambda$ is both open and continuous. 8

Unit – III

5. (a) Show that every second countable space is first countable but converse may not be true. 8

   (b) State and prove Lindelof theorem. Also show that converse of Lindelof theorem is not true. 8

6. (a) Define T_0 and T_1 –spaces and show that a top space is T_1 iff every subset consisting of exactly one point is closed. 8

   (b) Show that the property of being a T_2-space is a topological property. 8

Unit – IV

7. (a) Define a compact set. Show that an infinite set with cocountable topology is not compact. 8

   (b) Show that a top space is compact iff any family of closed sets having FIP has a non-empty intersection. 8

8. (a) Prove that every sequentially compact top space is countably compact. 8

   (b) Let (X*, j*) be a one point compactification of a non-compact top space (X,j), then prove that (X*,j*) is a Hausdorff space iff (X,j) is locally compact. 8

Unit – V

9.

   a) Show that the union of two or more topologies need not be a topology.
   b) Prove the $(A \cap B)^0 = A^0 \cap B^0$
   c) Define relative topology.
   d) Define open and closed mappings.
   e) Show that every closed subset of a compact space is compact.
   f) Show that cofinite topology is a T_0-space.
   g) Prove that every indiscrete space is locally compact.
   h) Define hereditary and topological properties.

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M.Sc. (Maths), First Semester Examination, Dec 2016 **Sr. No. 8168-A**

Real Analysis – I

Time: 3 Hours Max. Marks: 80

Note: Attempt five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) If $f_1 \epsilon R(\alpha)$ and $f_2 \epsilon R(\alpha)$, then $(f_1+f_2) \epsilon R(\alpha)$ and $\int_a^b (f_1+f_2)d\alpha = \int_a^b f_2 d\alpha$ 8

   (b) If $f \epsilon R(\alpha)$ on [a,b], m≤f≤M and ϕ is continuous on [m,M] and $h(x) = \phi(f(x))$ on [a,b]. Then prove that $h \in R(\alpha)$ on [a,b]. 8

2. (a) Show that $f \epsilon R(\alpha)$ on [a,b] iff for every $\varepsilon > 0$, there exists a partition P of [a,b] such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$ 8

   (b) State and prove second mean value theorem. 8

Unit – II

3. (a) State and prve Weirstrass M-test for unform convergence of a sequence. 8

   (b) If a sequence of Continuous functions converges uniformly to f, then prove that f is continuous. 8

4. (a) State and prove Dirichlet's test for uniform convergence of a series. 8

   (b) Show that the series $\sum f_n$ whose sum to n terms is $S_n(x) = hxe^{-nx^2}$ is pointwise but not uniformly convergent on any interval [0,k],k>0. 8

Unit – III

5. (a) Let $f(x,y) = \sqrt{x^4+y^4+1}$, then evaluate $f_x(1,2)$ and $f_y(1,2)$. 8

   (b) State and prove Young's theorem. 8

6. (a) Prove that by the transformation u=x-ct, v=x+ct, the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ reduces to $\frac{\partial^2 u}{\partial u \partial v} = 0$. 8

   (b) State and prove Implicit function Theorem. 8

Unit – IV

7. (a) State and prove uniqueness theorem for power series. 8

   (b) State and prove Taylor's Theorem. 8

8. (a) State and prove Stakes Theorem. 8

   (b) Let $\sum a_n x^n$ be a power series with finite radius of convergence R and let $f(x) = \sum a_n x^n, |x| < R$. If the series $\sum a_n x^n$ converges at end point $x = R$, then prove that $\lim_{n \to R^-} \sum a_n R^n$ 8

P.T.O.

Unit – V

9. 16

a) Define Partition of an interval.

b) Define rectifiable curve.

c) Define Pointwise and uniform convergence.

d) State Weirstrass approximation theorem.

e) State Schwarz Theorem.

f) Define Jacobian of u,v w.r.t. x,y.

g) State Tauber's Theorem.

h) Define Gamma Function.

155251

M.Sc. (Maths), First Semester Examination, Dec 2016 **Sr. No. 8022**

Topology – I

Time: 3 Hours

Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

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   (b) Prove that in a top space (X,j), $\overline{A} = A \cup d(A) \forall A \subseteq X$. 10
2. (a) Prove that a family β of sets is a base for a topology for the set $X = U\{B: B\epsilon\beta\}$ iff for every $B_1, B_2 \in \beta$ and every $x \in B_1 \cap B_2, \exists\ a\ B \in \beta$ s.t. $x \in \beta \underline{c} B_1 \cap B_2$. 8
   (b) Characterize topology in terms of Kuratowsti closure operator. 8

Unit – II

3. (a) If f is a mapping of a top space X into another top space, then prove that f is continuous on X iff $f(c(E)) \underline{C} C^* f(E) \forall E \underline{C} X$. 8
   (b) If a connected set C has a non-empty intersection with both a set E and the complement of E in a top space (X,j), then show that C has a non-empty entersection with the boundary of E. 8
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   (b) Prove that every protection $\pi_\lambda: X \to X_\lambda$ on a product space $X = {}^{\pi}_{\lambda} X_\lambda$ is both open and continuous. 8

Unit – III

5. (a) Show that every second countable space is first countable but converse may not be true. 8
   (b) State and prove Lindelof theorem. Also show that converse of Lindelof theorem is not true. 8
6. (a) Define T_0 and T_1 –spaces and show that a top space is T_1 iff every subset consisting of exactly one point is closed. 8
   (b) Show that the property of being a T_2-space is a topological property. 8

Unit – IV

7. (a) Define a compact set. Show that an infinite set with cocountable topology is not compact. 8
   (b) Show that a top space is compact iff any family of closed sets having FIP has a non-empty intersection. 8
8. (a) Prove that every sequentially compact top space is countably compact. 8
   (b) Let (X*, j*) be a one point compactification of a non-compact top space (X,j), then prove that (X*,j*) is a Hausdorff space iff (X,j) is locally compact. 8

Unit – V

a) Show that the union of two or more topologies need not be a topology.
b) Prove the $(A \cap B)^0 = A^0 \cap B^0$
c) Define relative topology.
d) Define open and closed mappings.
e) Show that every closed subset of a compact space is compact.
f) Show that cofinite topology is a T_0-space.
g) Prove that every indiscrete space is locally compact.
h) Define hereditary and topological properties.

M. Sc. (Mathematics)

Sr. No. 12068

II nd Sem. Examination, May 2017

Methods of Applied Mathematics

Time:- 3 Hours

Section - I

M.M:-80

1) a) Solve heat equation in cylinderical polar co-ordinates. (8)

b) Obtain axially-symmetrical solution of 3-D Laplace equation. (8)

2) a) Explain the concept of Gradience in curvi-Linear coordinates. (8)

b) Find divergent and curl in cylindrical coordinates (r, θ, z) where $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, $x_3 = z$ (8)

Section - II

3) Obtain the solution of free vibration of a large circular elastic membrane governed by initial value problem

$$c^2\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = \frac{\partial^2 u}{\partial t^2}, \quad 0 < r < \infty, \; t > 0,$$

$$u(r,0) = f(r), \quad u_t(r,0) = g(r) \text{ for } 0 \le r < \infty$$

where $c^2 = (T/\rho) =$ constant, T is the tension in membrane and ρ is surface density. (16)

4) a) Find n^{th} order Hankel transform of $f(r) = r^n \exp(-ar^2)$. (8)

b) Define relation between fourier and hankel transform. (8)

Section - III

5) State and prove Mellin inversion theorem. (16)

6) a) Define properties of hypergeometric functions. (8)

b) Find Mellin Transform of

(i) $\sin(t)$

(ii) e^{at} (8)

Section - IV

7) State and prove Branchistochrone problem. (16)

8) a) Define Euler's equation. (8)

b) Find the extremals of functional

$$\int_{x^0}^{x^1} (y^2 - y'^2 - 2y \sin x \cdot dx$$ (8)

Section- V

9)

a) Define Hankel transform.

b) What do you understand by isoperimetric problem.

c) Write down any two operational properties of hankel transform.

d) Define boundary value problem.

e) State Laplace equation in cylindrical polar co-ordinates.

f) Write heat equation for spherical polar co-ordinates.

g) Define functionals.

Sr. No. 6175-A

M.Sc. (Maths), Third Semester Examination, Dec 2016

Integral Equations

Time: 3 Hours | Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Show that the integral equation 8

$$g(s) = f(s) + \frac{1}{\pi}\int_0^{2\pi}[\sin(s+t)]\, g(t)dt$$

Possesses no solution for $f(s) = s$, but it possesses infinitely many solutions for $f(s) = 1$.

(b) Find the eigen values and eigen functions of the homogeneous integral equation 8

$$g(s) = \lambda \int_1^2 \left(st + \frac{1}{st}\right) g(t)dt$$

2. State and prove Fredholm Theorem. 16

Unit – II

3. (a) Solve the integral equation $g(s) = 1 + \lambda \int_1^{\pi}[\sin\,(s+t)]g(t)dt$ 8

(b) Prove that the resolvent kernel for a Volterra integral equation of the second kind is an entire function of λ for any given (s,t). 8

4. State and prove Fredholm's third Theorem. 16

Unit – III

5. (a) Solve the symmetric integral equation $g(s) = f(s) + \lambda \int_{-1}^{1}(st + s^2t^2)g(t)dt$ where $f(s) = (s+1)^2$ 8

(b) Prove that a symmetric L_2-Kerner having a finite number of eigenvalues is degenerate kernel. 8

6. State and prove Hilbert-Schmidt Theorem. 16

Unit – IV

7. (a) Obtain the solution of integral equation 8

$$f(s) = \frac{2s^{2n}}{\Gamma(\alpha)}\int_s^{\infty}(t^2 - s^2)^{\alpha-1}t^{-2\alpha-2n+1}g(t)dt, \quad 0 < \alpha < 1$$

(b) Write a note on Cauchy's principal value for integrals solution of the Cauchy type singular integral equation. 8

8. Solve the singular integral equation of first kind 16

$$f(x) = \frac{v}{2\pi}\int_0^{2\pi} \phi(\xi) \cot\left(\frac{\xi - x}{2}\right) d\xi \qquad \text{where } \xi \text{ is a constant.}$$

Unit – V

9. 16

a) Define Eigen functions
b) Define convolution integral
c) Solve $g(s) = f(s) + \lambda \int_0^s e^{s-t} g(t)dt$
d) State Fredhalm's first theorm.
e) Define orthonormal system of functions.
f) Prove that eigenvalues of a symmetric kernel are real.
g) Define Abel integral equation.
h) Define Hilbert kernel.

Sr. No. 6171-A

M.Sc. (Maths), Third Semester Examination, Dec 2016

Functional Analysis

Time: 3 Hours Max. Marks: 80

Note: Attempt five questions in all, selecting one question from each unit. Question No. 1 is compulsory.

Q.1 Short answer type questions:

2x8=16

i) Define Normed linear space with example.

ii) Define Dual space with example.

iii) Let N be Normed linear space; x,y are two distinct vectors in N, than there exists a functional F on N such the F(x-y)=||x-y||

iv) Define weak and strong Convergence.

v) State open Mapping theorem.

vi) Prove that $|<x,y>| \leq ||x||\ ||y||$

vii) State and prove Bessel's Inequality.

viii) Define sesquilinear function.

UNIT – I

Q.2 Prove that for $1 \leq p < \infty$, L^p - space is a complete normed linear space 16

Q.3 If N and N^1 are Normed linear spaces, than the set $B(N,N^1)$ of all continuous linear transformation of N into N^1 is itself a normed linear space with respect to the pointwise linear operations and the Norm defined by $||T||=Sup\{||Tx||:||x||\leq l\}$. Further if N^1 is Banach space, then $B(N,N^1)$ is also a Banach space. 16

UNIT – II

Q.4 If M is a linear subspace of Normed Linear space N and f is a functional defined on M then f can be extended to another functional F in N* such that ||f||=||F||. 16

Q.5 (a) Let B is a Banach space and N is Normed linear space. If $\{T_i\}$ is a non-empty set of continuous linear transformation of B into N with property that the set $\{T_i(x)\}$ is a bounded subset of N for each vector x in B then $\{||T_i||\}$ is a bounded set of Numbers. 8

(b) Show that the Normed space of all polynomials with norm defined by $||x|| = \max_j |\propto_j|$ (where $\propto_j$'s one coefficients of polynomial) is not complete. 8

UNIT - III

Q.6 (a) Prove that in a Hilbert space $x_n \overset{w}{\rightarrow} x$ iff $< x_n, z > \rightarrow < x, z >$ $\forall z$ in the space. 8

(b) In a normed space, we have $x_n \overset{w}{\rightarrow} x$ iff 8

i) the sequence $(||x_n||)$ is bounded

ii) for every element f of a closed subset MCX^1, we have $f(x_n) \rightarrow f(x)$

Q.7 (a) If W is a subspace of finite dimensional inner product space than prove that $V=W\oplus W^{\perp}$ 8

(b) Prove that $|(u,v)|=||u||\ ||v||$ iff u, v are linearly dependent. 8

UNIT - IV

Q.8 (a) If y is a closed subspace of a Hilbert space then $y=y^{\perp\perp}$ 8

(b) Let $T:H\rightarrow H$ be a bounded self adjoint linear operator on a complex Hilbert space H. Then 8

i) all the eigen values of T are real.

ii) eigen vectors corresponding to different eigen values of T are orthogonal.

Q.9 State and prove Riesz representation theorem for bounded linear functionals on a Hilbert space. 16

Sr. No. 9020

M.Sc. (Maths), Third Semester Examination, Dec 2016

Integral Equations & Calculus of Variations

Time: 3 Hours Max. Marks: 80

Note: Attempt one question from each of four units. Unit V is compulsory.

Unit – I

1. (a) Define integral equations. Also describe the various types of linear integral equations. 8

   (b) Reduce the IVP 8

   y"(x)+A(x)y'(x)+B(x)y(x)=f(x)'

   $y(a) = q_0, y'(a) = q_1, a \leq x \leq b$ to the Volteera IE.

2. (a) Find the Newmann series for the solution of the IE $y(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2} y(t)dt$ 8

   (b) Solve the generalized Abel's IE $f(x) = \int_0^x \frac{y(t)dt}{(x-t)^\alpha}, 0 < \alpha < 1$ 8

Unit – II

3. (a) Explain the method of successive approximation for the solution of Fredholm IE. 8

   (b) Solve $y(x) = f(x) + \lambda \int_0^1 xe^t y(t)dt$ by the method of Fredholm determinants 8

4. (a) Show that the IE $y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)y(t)dt$ possesses no solution for $f(x) = x$. 8

   (b) Find the approximate solution to the Fredholm IE $y(x) = x + \int_{-1}^1 e^{xt}\, y(t)dt$ by considering only the first two terms of e^{xt}. 8

Unit – III

5. (a) Solve $y'' = -x, y(0) = 0, y(1) = 0$ using Green's function and verify the answer. 8

   (b) Reduce the BVP $y'' + xy = 1, y(0) = 0, y(l) = 1$ into IE using Green's function. 8

6. (a) State and prove Hilbert Schmidt theorem. 8

   (b) Find the e-values and e-functions of the homogeneous equation $y(x) = \lambda \int_0^1 K(x,t)\, y(t)$ with symmetric keenel $K(x,t) = \begin{cases} x(1-t), & x < t \\ t(1-x), & x > t \end{cases}$ 8

Unit – IV

7. (a) Find the curve passing through (x_0, y_0) and (x_1, y_1) which generates the surface of minimum area when rotated about x-axes. 8

   (b) Find the extremals of the functional $\int_{-a}^{a}(Py + \frac{1}{2}uy''^2)dx$, which satisfies the boundary conditions $y(-a) = 0, y(a) = 0, y'(-a) = 0, y'(a) = 0$ 8

8. (a) Prove that the sphere is a solid figure of revolution which for a given surface area has a maximum volume. 8

   (b) Find the geodesies on a right circuler cylinder of radius a. 8

Unit – V

9. 16

   a) Mention the different types of keenels.
   b) State Fredholm first theorem.
   c) Explain approximate method for the solution of Fredholm IE.
   d) State four basic properties of Green's function.
   e) Reduce VIE of first kind $x = \int_0^x 3^{x-t} y(t)dt$ into second kind.
   f) State any two properties of e-values and e-functions of a symmetric keenel.
   g) Distinguish between functions and functional.
   h) Define Isoperimetric problems. Also provide suitable examples.

Sr. No. 9019

M.Sc. (Maths), Third Semester Examination, Dec 2016

Mathematical Statistics

Time: 3 Hours Max. Marks: 80

Note: Attempt any five questions in all selecting one question from each unit. Unit V is compulsory. All questions carry equal marks.

Unit – I

1. (a) Give classical definition of probability. A bag contains 4 white, 5 red and 6 green balls. Three balls are drawn at random. What is the probability that a white, a red and a green ball are drawn?
   (b) State and prove addition theorem of probabilities.
2. (a) State and prove Bayes' theorem on probability.
   (b) A bag contains 6 white and 9 black balls. Four balls are drawn at a time. Find the probability for the first draw to give 4 white and the second to give 4 black balls in each of the following cases:
   i) The balls are replaced before the second draw.
   ii) The balls are not replaced before the second draw.

Unit – II

3. (a) Explain the following: Sample Space, distribution function, Discrete and Continuous random variables alongwith one example for each.
   (b) A random variable X has the following probability distribution:

X:	0	1	2	3	4	5	6	7	8
f(x):	K	3K	5K	7K	9K	11K	13K	15K	17K

   Find the value of K; $P(X<3)$; $P(X\geq3)$; $P(0<X<5)$. Also obtain mean of the distribution.

4. (a) Given the following bivariate probability distribution, obtain (i) marginal distributions of X and Y. (ii) Conditional distribution of X given Y=2.

Y \ X	-1	0	1
0	1/15	2/15	1/15
1	3/15	2/15	1/15
2	2/15	1/15	2/15

   (b) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them.
   (c) Prove that the moment generating function of the sum of a number of independent random variables is rqual to the product of their respective moment generating functions.

Unit – III

5. (a) Define Binomial distribution and prove the following relation for this distribution:
   $$\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d}{dp}\mu_r\right]$$
   (b) Find moment generating and cumulant generating functions for Poisson's distribution. Also show that all cumulants are equal for Poisson's distribution.
6. (a) Prove that for a Normal distribution:
   $$\mu_{2n+1} = 0 \text{ and } \mu_{2n} = 1.3.5 \ldots..(2n-1)\sigma^{2n}$$
   (b) Define exponential distribution. Obtain its mean, variance and moment generating function.

P.T.O.

Unit – IV

7. (a) What do you understand by estimation?
   (b) Write short notes on the following:
      i) Simple and Composite Hypothesis
      ii) Two types of errors.
      iii) Standard error of estimate.

8. (a) A dice is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the dice cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.
   (b) A sample of 900 members has a mean 3.4 cms and S.D. 2.61 cms. Is the sample from a large population of mean 3.25cms and S.D. 2.61 cms? If the population is normal and its mean is unknown, find the 95% and 98% fiducial limits of true mean.

Unit – V

9.
   a) Define mutually exclusive events and equally likely events.
   b) Find the probability of throwing a number greater than 2 with an ordinary dice?
   c) Define geometric distribution.
   d) Find the expectation of the number on a dice when thrown.
   e) Explain moment generating function. Why is it called so?
   f) Obtain M.G.F. for a Normal distribution.
   g) Define parameter & statistic.
   h) State Central limit theorem.